TRINITY COLLEGE

IN NOMINE DOMINI

Mathematics Specialist Units 1,2
Test 4 2018

Section 1 Calculator Free

Trigonometry

STUDENT'S NAME SOLUTIONS

DATE: Thursday 26 July **TIME:** 28 minutes **MARKS:** 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Determine the exact value of $\cos 105°$.

$$\cos(60° + 45°)$$

$$= \cos 60° \cos 45° - \sin 60° \sin 45°$$

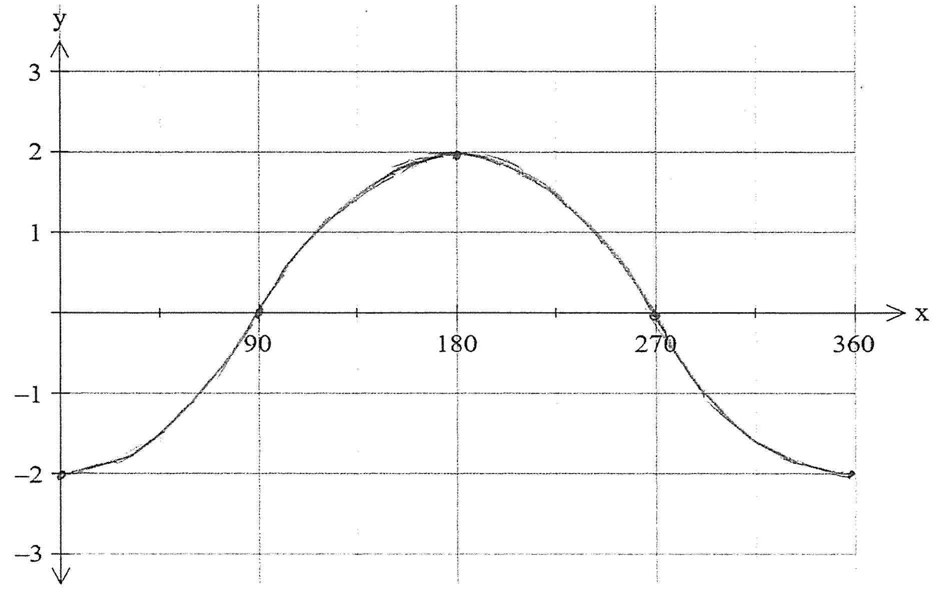
$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

2. (9 marks)

(a) For the function $y = 2\sin(x - 90^{\circ})$

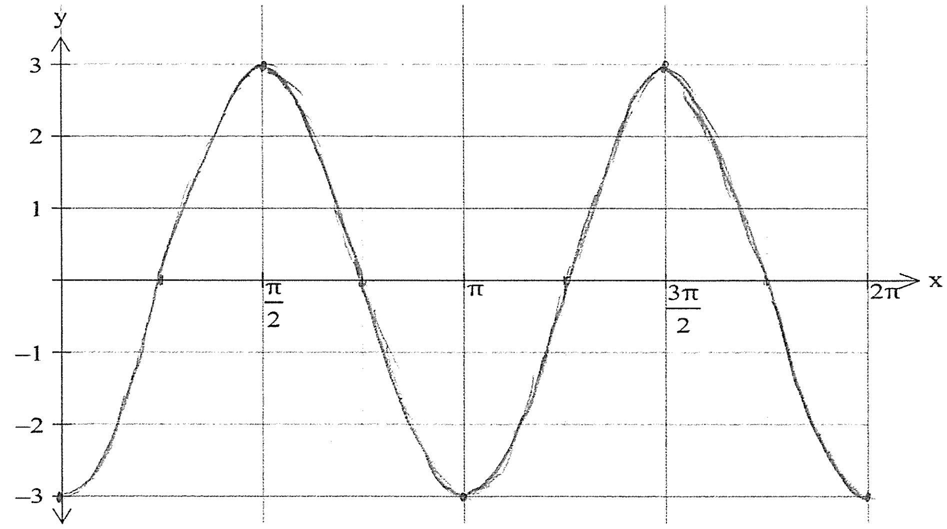
(i) sketch the function on the axes below. [2]



(ii) determine the amplitude and change of phase. [2]

(b) For the function $y = -3\cos 2x$

(i) sketch the function on the axes below. [3]



(ii) determine the amplitude and period. [2]

3. (3 marks)

Prove $\cot\theta(\cos\theta - \sec\theta) = -\sin\theta$

$$
\begin{aligned}
LHS &= \frac{\cos\theta}{\sin\theta}\left(\cos\theta - \frac{1}{\cos\theta}\right) \\
&= \frac{\cancel{\cos\theta}}{\sin\theta}\left(\frac{\cos^2\theta - 1}{\cancel{\cos\theta}}\right) \\
&= -\frac{\sin^2\theta}{\sin\theta} \\
&= -\sin\theta \\
&= RHS
\end{aligned}
$$

4. (9 marks)

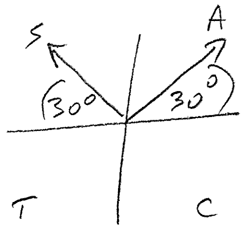
(a) Solve $2\sin x\cos x = \cos x$ $\qquad -180° \le x \le 180°$ [4]

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\qquad \sin x = \frac{1}{2}$

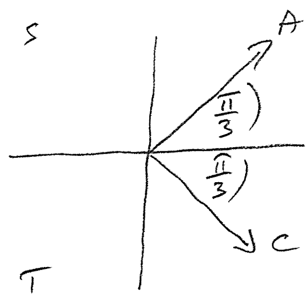
$x = 90°, -90°$ $\qquad x = 30°, 150°$



(b) $\cos 2x \cos\frac{\pi}{6} - \sin 2x \sin\frac{\pi}{6} = 0.5$ $\qquad 0 \le x \le 2\pi$ [5]

$\cos\left(2x + \frac{\pi}{6}\right) = 0.5$

$2x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$



$2x + \frac{\pi}{6} = \frac{\pi}{3}$

$2x = \frac{\pi}{6}$

$x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{21\pi}{12}$

5. (5 marks)

Solve $2\cos^2\theta - 7\cos\theta - 4 = 0$ $\quad\theta$ *radians*

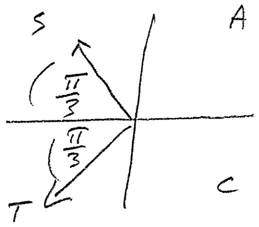
$$(2\cos\theta + 1)(\cos\theta - 4) = 0$$

$2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$\cos\theta = 4$

NO SOLN.



$$\theta = \begin{cases} \frac{2\pi}{3} + 2n\pi \\ \frac{4\pi}{3} + 2n\pi \end{cases} \qquad n \in \mathbb{Z}$$

TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 4 2018

Section 2 Calculator Assumed
Trigonometry

STUDENT'S NAME ______________________________

DATE: Thursday 26 July **TIME:** 25 minutes **MARKS:** 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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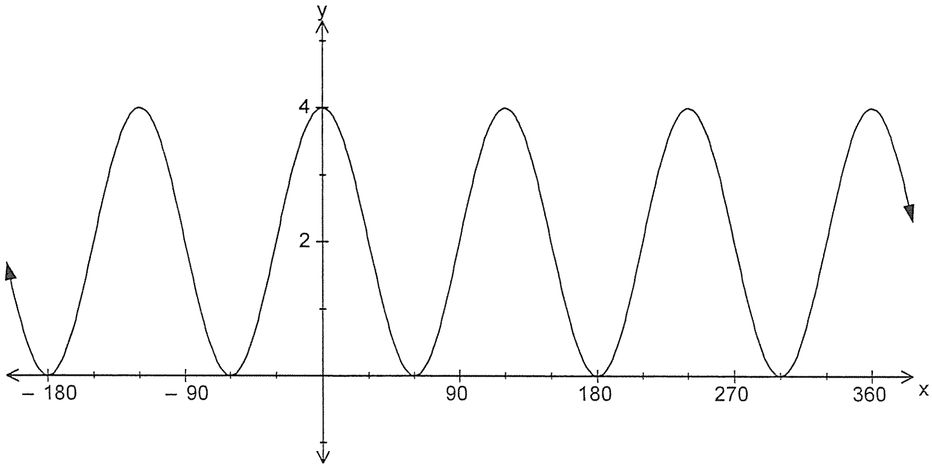
6. (2 marks)

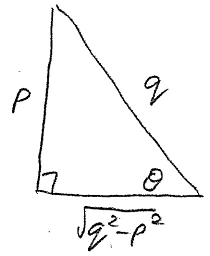
Determine the equation of the function shown below.



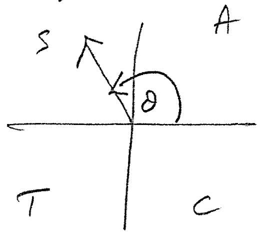
$y = 2\cos(3x) + 2$

7. (7 marks)

Given $\sin\theta = \dfrac{p}{q}$ where $\dfrac{\pi}{2} < \theta < \pi$, determine



(a) $\tan\theta$ $\quad -\dfrac{p}{\sqrt{q^2-p^2}}$ [2]



(b) $\sin 2\theta$ $= 2\sin\theta\cos\theta$ [2]

$= 2 \times \dfrac{p}{q} \times \left(-\dfrac{\sqrt{q^2-p^2}}{q}\right)$

$= \dfrac{-2p\sqrt{q^2-p^2}}{q^2}$

(c) $\cos\dfrac{\theta}{2}$ $\quad \cos\theta = 2\cos^2\dfrac{\theta}{2} - 1$ [3]

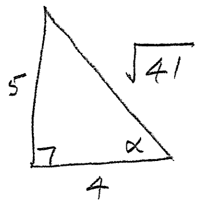
$\pm\sqrt{\dfrac{\cos\theta + 1}{2}} = \cos\dfrac{\theta}{2}$

$\dfrac{\theta}{2}$ IN 1^{ST} QUADRANT $\therefore \cos\dfrac{\theta}{2}$ POSITIVE

$\sqrt{\dfrac{\dfrac{\sqrt{q^2-p^2}}{q} + 1}{2}} = \cos\dfrac{\theta}{2}$

8. (9 marks)

(a) Express $4\cos x - 5\sin x$ in the form $R\cos(x+\alpha)$ [3]



$= \sqrt{41}\left(\frac{4}{\sqrt{41}}\cos x - \frac{5}{\sqrt{41}}\sin x\right)$

$= \sqrt{41}\left(\cos x \cos\alpha - \sin x \sin\alpha\right)$

$= \sqrt{41}\cos\left(x + 51.3^\circ\right)$

$\tan\alpha = \frac{5}{4}$

$\alpha = 51.3^\circ$

$\left[\alpha = 0.90 \qquad \sqrt{41}\cos(x + 0.90)\right]$

(b) Determine the maximum value of $4\cos x - 5\sin x$ and the smallest positive value of x when the maximum value occurs. [3]

MAX VALUE $= \sqrt{41}$ (MAX VALUE COS $= 1$)

$\cos\theta = 1$

$\theta = 360^\circ$

$x + 51.3^\circ = 360^\circ$

$x = 308.7^\circ$

$\theta = 2\pi$

$x + 0.9 = 2\pi$

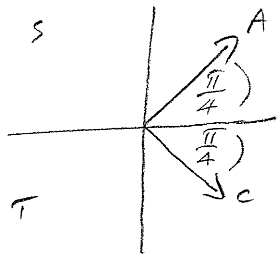
$x = 5.38$

(c) Solve $4\cos x - 5\sin x = \sqrt{20.5}$ for $0 \le x \le 2\pi$ [3]

$\sqrt{41}\cos(x + 0.9) = \sqrt{20.5}$

$\cos(x + 0.9) = \frac{\sqrt{20.5}}{\sqrt{41}}$

$\cos(x + 0.9) = \frac{1}{\sqrt{2}}$



$x + 0.9 = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

$x = \cancel{-0.11}, 4.6, 6.2$

9. (7 marks)

(a) Prove $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$ [3]

$$LHS = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$$

$$= \frac{\frac{\cos 2x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$$

$$= \cos 2x = RHS$$

(b) Hence, or otherwise, show that if $\cos 2\alpha = \tan^2 \beta$ then $\cos 2\beta = \tan^2 \alpha$. [4]

$$\cos 2\beta = \frac{1-\tan^2\beta}{1+\tan^2\beta}$$

$$= \frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

$$= \frac{1-(1-2\sin^2\alpha)}{1+(2\cos^2\alpha - 1)}$$

$$= \frac{2\sin^2\alpha}{2\cos^2\alpha}$$

$$= \tan^2\alpha$$